Fórmulas, Expansões e Conceitos Essenciais

- **Produtos Notáveis e Fatoração:**
  - $(a \pm b)^2 = a^2 \pm 2ab + b^2$
  - $a^2 - b^2 = (a+b)(a-b)$
  - $(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$
  - $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
  - $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
  - $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$

- **Operações com Frações:** $(b, d \neq 0)$
  - Adição/Subtração: $\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$
  - Multiplicação: $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$
  - Divisão: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc} \quad (c \neq 0)$

- **Equação do 2º Grau ($ax^2 + bx + c = 0, a \neq 0$):** Fórmula de Bhaskara: $x = \frac{-b \pm \sqrt{\Delta}}{2a}$, onde $\Delta = b^2 - 4ac$.
  Soma das raízes (S): $x_1 + x_2 = -b/a$. Produto das raízes (P): $x_1 x_2 = c/a$.

- **Logaritmos:** (Base $b > 0, b \neq 1$; Argumentos $M > 0, N > 0$)
  - Definição: $\log_b M = x \iff b^x = M$
  - Consequências da definição: $\log_b 1 = 0$; $\log_b b = 1$; $\log_b b^k = k$; $b^{\log_b M} = M$.
  - Produto: $\log_b(MN) = \log_b M + \log_b N$
  - Quociente: $\log_b(M/N) = \log_b M - \log_b N$
  - Potência: $\log_b(M^k) = k \cdot \log_b M$
  - Mudança de Base: $\log_b M = \frac{\log_c M}{\log_c b}$ (Base $c > 0, c \neq 1$)

- **Trigonometria Detalhada:**
  - **Relações Trigonométricas Principais**
    - **Identidade Fundamental:**

      $\sin^2\theta + \cos^2\theta = 1$

      Outras Relações Fundamentais:

      $$\tan\theta = \frac{\sin\theta}{\cos\theta}; \quad \cot\theta = \frac{\cos\theta}{\sin\theta}; \quad \sec\theta = \frac{1}{\cos\theta}; \quad \csc\theta = \frac{1}{\sin\theta}$$

      $1 + \tan^2\theta = \sec^2\theta; \quad 1 + \cot^2\theta = \csc^2\theta$

* **Adição e Subtração de Ângulos:**

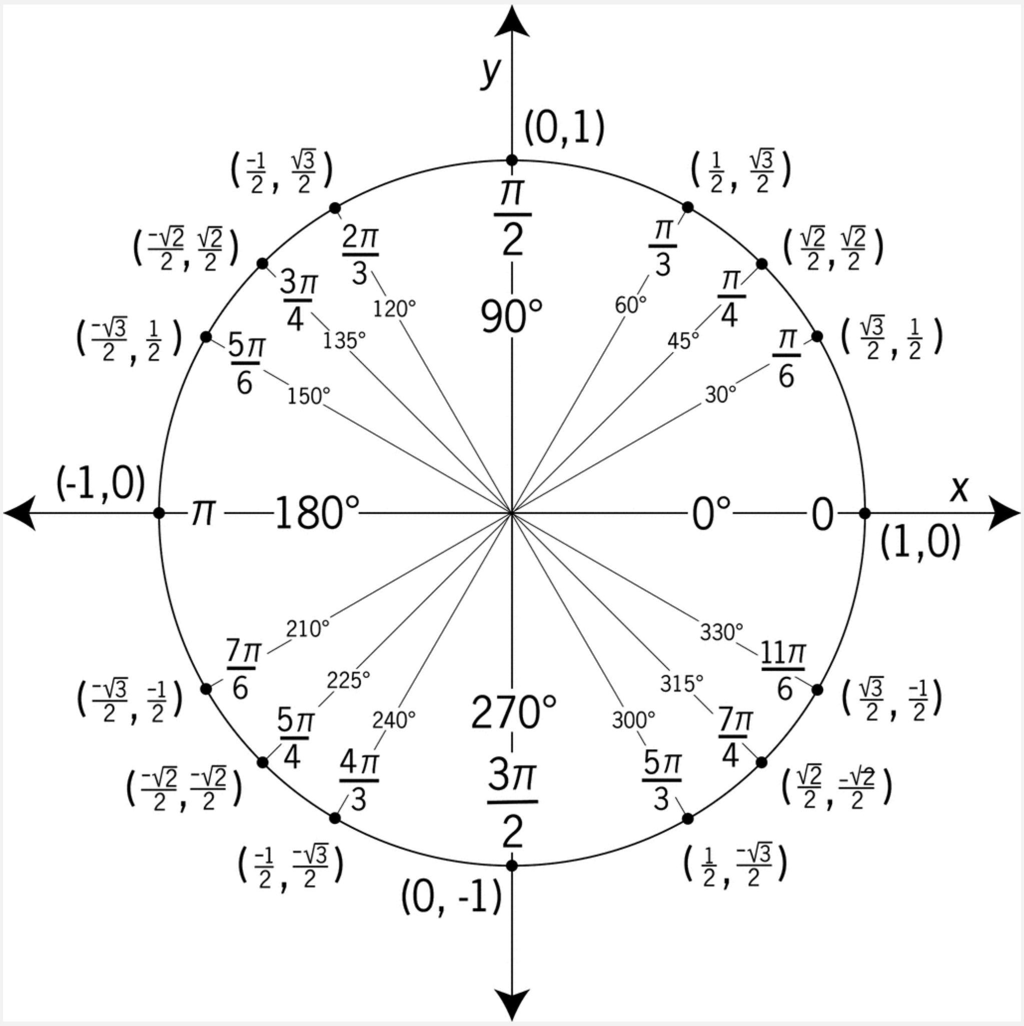
$$\begin{cases} \sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta, \\ \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta, \\ \tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta} \end{cases}$$

* **Ângulo Duplo:**

$$\begin{cases} \sin 2\theta = 2\sin\theta\cos\theta, \\ \cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta, \\ \tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta} \end{cases}$$

* **Ângulo Metade (ou Fórmulas de Potência Reduzida):**

$$\begin{cases} \sin^2\left(\frac{\theta}{2}\right) = \dfrac{1 - \cos\theta}{2} \implies \sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\dfrac{1 - \cos\theta}{2}}, \\ \cos^2\left(\frac{\theta}{2}\right) = \dfrac{1 + \cos\theta}{2} \implies \cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\dfrac{1 + \cos\theta}{2}}, \\ \tan\left(\frac{\theta}{2}\right) = \dfrac{\sin\theta}{1 + \cos\theta} = \dfrac{1 - \cos\theta}{\sin\theta} \end{cases}$$



y
x
(0,1)
(-1,0)
(1,0)
(0, -1)
0°
90°
180°
270°
30°
45°
60°
120°
135°
150°
210°
225°
240°
300°
315°
330°
0
$\frac{\pi}{6}$
$\frac{\pi}{4}$
$\frac{\pi}{3}$
$\frac{\pi}{2}$
$\frac{2\pi}{3}$
$\frac{3\pi}{4}$
$\frac{5\pi}{6}$
π
$\frac{7\pi}{6}$
$\frac{5\pi}{4}$
$\frac{4\pi}{3}$
$\frac{3\pi}{2}$
$\frac{5\pi}{3}$
$\frac{7\pi}{4}$
$\frac{11\pi}{6}$
$(\frac{\sqrt{3}}{2}, \frac{1}{2})$
$(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$
$(\frac{1}{2}, \frac{\sqrt{3}}{2})$
$(\frac{-1}{2}, \frac{\sqrt{3}}{2})$
$(\frac{-\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$
$(\frac{-\sqrt{3}}{2}, \frac{1}{2})$
$(\frac{-\sqrt{3}}{2}, \frac{-1}{2})$
$(\frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2})$
$(\frac{-1}{2}, \frac{-\sqrt{3}}{2})$
$(\frac{1}{2}, \frac{-\sqrt{3}}{2})$
$(\frac{\sqrt{2}}{2}, \frac{-\sqrt{2}}{2})$
$(\frac{\sqrt{3}}{2}, \frac{-1}{2})$

θ **(gr)**	θ **(rd)**	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
$0°$	0	0	1	0	indef.	1	indef.
$15°$	$\frac{\pi}{12}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$2-\sqrt{3}$	$2+\sqrt{3}$	$\sqrt{6}-\sqrt{2}$	$\sqrt{6}+\sqrt{2}$
$30°$	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2
$45°$	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
$60°$	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$
$75°$	$\frac{5\pi}{12}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$2+\sqrt{3}$	$2-\sqrt{3}$	$\sqrt{6}+\sqrt{2}$	$\sqrt{6}-\sqrt{2}$
$90°$	$\frac{\pi}{2}$	1	0	indef.	0	indef.	1
$105°$	$\frac{7\pi}{12}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$-\frac{\sqrt{6}-\sqrt{2}}{4}$	$-(2+\sqrt{3})$	$-(2-\sqrt{3})$	$-(\sqrt{6}+\sqrt{2})$	$\sqrt{6}-\sqrt{2}$
$120°$	$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$	-2	$\frac{2\sqrt{3}}{3}$
$135°$	$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1	-1	$-\sqrt{2}$	$\sqrt{2}$
$150°$	$\frac{5\pi}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$	$-\frac{2\sqrt{3}}{3}$	2
$165°$	$\frac{11\pi}{12}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$-\frac{\sqrt{6}+\sqrt{2}}{4}$	$-(2-\sqrt{3})$	$-(2+\sqrt{3})$	$-(\sqrt{6}-\sqrt{2})$	$\sqrt{6}+\sqrt{2}$
$180°$	π	0	-1	0	indef.	-1	indef.
$195°$	$\frac{13\pi}{12}$	$-\frac{\sqrt{6}-\sqrt{2}}{4}$	$-\frac{\sqrt{6}+\sqrt{2}}{4}$	$2-\sqrt{3}$	$2+\sqrt{3}$	$-(\sqrt{6}-\sqrt{2})$	$-(\sqrt{6}+\sqrt{2})$
$210°$	$\frac{7\pi}{6}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$-\frac{2\sqrt{3}}{3}$	-2
$225°$	$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1	1	$-\sqrt{2}$	$-\sqrt{2}$
$240°$	$\frac{4\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	-2	$-\frac{2\sqrt{3}}{3}$
$255°$	$\frac{17\pi}{12}$	$-\frac{\sqrt{6}+\sqrt{2}}{4}$	$-\frac{\sqrt{6}-\sqrt{2}}{4}$	$2+\sqrt{3}$	$2-\sqrt{3}$	$-(\sqrt{6}+\sqrt{2})$	$-(\sqrt{6}-\sqrt{2})$
$270°$	$\frac{3\pi}{2}$	-1	0	indef.	0	indef.	-1
$285°$	$\frac{19\pi}{12}$	$-\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$-(2+\sqrt{3})$	$-(2-\sqrt{3})$	$\sqrt{6}+\sqrt{2}$	$-(\sqrt{6}-\sqrt{2})$
$300°$	$\frac{5\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$	2	$-\frac{2\sqrt{3}}{3}$
$315°$	$\frac{7\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1	-1	$\sqrt{2}$	$-\sqrt{2}$
$330°$	$\frac{11\pi}{6}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	-2
$345°$	$\frac{23\pi}{12}$	$-\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$-(2-\sqrt{3})$	$-(2+\sqrt{3})$	$\sqrt{6}-\sqrt{2}$	$-(\sqrt{6}+\sqrt{2})$
$360°$	2π	0	1	0	indef.	1	indef.

- **Derivadas - Definição e Regras Básicas:**
  - Definição: $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ ou $f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$.
  - $(c)' = 0$ (c = constante)
  - $(x^n)' = nx^{n-1}$
  - $(cf(x))' = cf'(x)$
  - $(f(x) \pm g(x))' = f'(x) \pm g'(x)$
  - Regra do Produto: $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$
  - Regra do Quociente: $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x)-f(x)g'(x)}{(g(x))^2}$
  - Regra da Cadeia: Se $y = f(u)$ e $u = g(x)$, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$. Ou $[f(g(x))]' = f'(g(x)) \cdot g'(x)$.

- **Derivadas - Funções Elementares:**
  - $(e^x)' = e^x$; $(a^x)' = a^x \ln a$
  - $(\ln|x|)' = \frac{1}{x}$; $(\log_b|x|)' = \frac{1}{x \ln b}$
  - $(\sin x)' = \cos x$; $(\cos x)' = -\sin x$; $(\tan x)' = \sec^2 x$
  - $(\cot x)' = -\csc^2 x$; $(\sec x)' = \sec x \tan x$; $(\csc x)' = -\csc x \cot x$
  - $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$; $(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$; $(\arctan x)' = \frac{1}{1+x^2}$

- **Integrais - Propriedades e Primitivas Básicas:**
  - $\int kf(x)dx = k\int f(x)dx$
  - $\int (f(x) \pm g(x))dx = \int f(x)dx \pm \int g(x)dx$
  - $\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$
  - $\int \frac{1}{x}dx = \ln|x| + C$
  - $\int e^x dx = e^x + C$; $\int a^x dx = \frac{a^x}{\ln a} + C$
  - $\int \sin x dx = -\cos x + C$; $\int \cos x dx = \sin x + C$
  - $\int \sec^2 x dx = \tan x + C$; $\int \csc^2 x dx = -\cot x + C$
  - $\int \sec x \tan x dx = \sec x + C$; $\int \csc x \cot x dx = -\csc x + C$
  - $\int \tan x dx = \ln|\sec x| + C = -\ln|\cos x| + C$
  - $\int \cot x dx = \ln|\sin x| + C$
  - $\int \sec x dx = \ln|\sec x + \tan x| + C$
  - $\int \csc x dx = \ln|\csc x - \cot x| + C$
  - $\int \frac{1}{\sqrt{a^2-x^2}}dx = \arcsin\left(\frac{x}{a}\right) + C$

- $\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + C$
- $\int \frac{1}{x\sqrt{x^2-a^2}} dx = \frac{1}{a} \text{arcsec}\left|\frac{x}{a}\right| + C$

- **Integrais - Método da Substituição:** Se $u = g(x)$, então $du = g'(x)dx$.

$$\int f(g(x))g'(x)dx = \int f(u)du$$

- **Integrais - Integração por Partes:**

$$\int udv = uv - \int vdu$$

(Escolha de u sugerida: Log.Inv.Alg.Trig.Exp)

- **Integrais - Substituições Trigonométricas:** (para $a > 0$)
  - Para $\sqrt{a^2 - x^2}$: $x = a\sin\theta, dx = a\cos\theta d\theta \Rightarrow \sqrt{a^2 - x^2} = a\cos\theta$ (para $\theta \in [-\pi/2, \pi/2]$).
  - Para $\sqrt{a^2 + x^2}$: $x = a\tan\theta, dx = a\sec^2\theta d\theta \Rightarrow \sqrt{a^2 + x^2} = a\sec\theta$ (para $\theta \in (-\pi/2, \pi/2)$).
  - Para $\sqrt{x^2 - a^2}$: $x = a\sec\theta, dx = a\sec\theta\tan\theta d\theta \Rightarrow \sqrt{x^2 - a^2} = a|\tan\theta|$. (Se $x \geq a$, $\theta \in [0, \pi/2) \Rightarrow a\tan\theta$; se $x \leq -a$, $\theta \in (\pi/2, \pi] \Rightarrow -a\tan\theta$).

- **Teorema Fundamental do Cálculo (TFC):**
  - Parte I: Se f é contínua em $[a, b]$, $g(x) = \int_a^x f(t)dt \Rightarrow g'(x) = f(x)$.
  - Parte II: Se f é contínua em $[a, b]$ e $F'(x) = f(x)$, então:

$$\int_a^b f(x)dx = F(b) - F(a) = [F(x)]_a^b$$